Equilibrium Configuration and Tensions of a Flexible Cable in a Uniform Flowfield

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Equations are developed from which the equilibrium configuration and tension of a flexible cable in a uniform flowfield may be determined. These equations are then used to demonstrate that the widely accepted assumption that the cable's tangential drag force is negligible is incorrect. The effect of the cable's weight is also demonstrated.

Introduction

IN the recent past a literature survey was made on the general area of towed vehicle systems by Genin.[1] Briefly, the conclusion of the survey is that much work remains to be done (both theoretically and experimentally) before a significantly greater level of understanding than currently exists is achieved in the area of towed vehicles.

One of the odd features of this topic area is that relatively few of the research reports written on towed vehicles have been published in journals. This feature has resulted in vast duplication of effort, especially in the last five years, during which time agencies of several governments have been supporting research efforts in the area of the stability of towed vehicle systems. Most researchers tend to duplicate (both by intent and inadverently) the works of Glauert,[2,3] who published his papers on the stability of an airborne, nonlifting towed vehicle in rectilinear motion in 1930 and 1934, respectively. Of the many investigations which followed Glauert's work, the most significant contribution was made in 1947 by L. Landweber and M. H. Protter.[4] It is equally significant that this paper has gone completely unnoticed by subsequent researchers. The main contribution of Ref. 4 is that the authors included the tangential friction force of the towing cable in their mathematical model. Some completely independent studies have been made in recent years on this problem but they fail to achieve the elegance of Ref. 4 and consequently never demonstrate the importance of the tangential drag. In general, the problem of the precise roll of the tangential drag has been of most concern to those contemplating experimental endeavors. See Ref. 5, for example.

With a computer solution in mind, in this paper we derive the basic equations governing the equilibrium and tension of a flexible cable and present them in a very simple form. Equally important, we illustrate the results, using measured quantities for the lift and drag forces, for a towed vehicle system (in air) considering both nylon and steel cables.

Tow Vehicle System

In this section we develop the results of our study of the effect of a cable's tangential drag force and weight on the orientation of a tow vehicle system, demonstrating that the skin-friction force is not a negligible effect as most investigators, sans proof, have assumed. The equations developed should prove to be a valuable starting point in future investigations to determine stability criteria for the system, and equally important, they can be used by practicing engineers to obtain more accurate estimates of cable shapes and tensions.

The assumptions made in this study were steady-state aerodynamics, constant tow vehicle velocity, equilibrium configuration for the tow cable (hence system), and the airplane and towed vehicle were assumed to be rigid bodies. The last assumption allowed for constant values of lift and drag to be employed for the towed vehicle when in straight flight at a constant speed. These assumptions are discussed at length in many of the reports listed in the references and will not be repeated herein.

Figure 1 is a sketch of the system analyzed, defining a few of the variables which appear in the development below. A free body diagram of a generic segment of the cable of length ds is shown in Fig. 2. Summing forces in directions tangent and normal to the cable at point A, we obtain after making a small-angle assumption on $d\varphi$

$$dw \cos\varphi + d\tau = dT \tag{1}$$

and

$$dw \sin\varphi + Td\varphi - dN = 0 \tag{2}$$

Let us write the tangential drag per unit length as

$$d\tau = f(\varphi)ds \tag{3}$$

the normal component per unit length as

$$dN = g(\varphi)dS \tag{4}$$

and the weight per unit length as

$$dw = \mu dS \tag{5}$$

where $f(\varphi)$ and $g(\varphi)$ are experimentally determined and are functions of the velocity of the cable, and μ is the cable's density times its cross-sectional area per unit length of cable. It turns out that $g(\varphi)$ can be expressed as

$$g(\varphi) = K \cos^2\varphi$$

where K, of course, is a function of the velocity of the cable. Thus, Eq. (4) may be rewritten as

$$dN = K \cos^2\varphi dS \tag{6}$$

It facilitates computations to leave (3) in its present form for now.

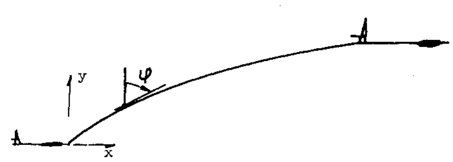

Fig. 1 Tow vehicle system.

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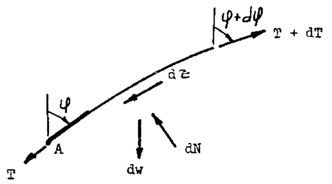

Fig. 2 Free body diagram.

We will now formulate the governing differential equations for the determination of cable shapes and tensions. Placing (3) and (5) into (1) and rearranging terms we obtain

$$dT/dS = f(\varphi) + \mu \cos\varphi \tag{7}$$

Placing (3) and (6) into (2) and rearranging terms we obtain

$$d\varphi/dS = (1/T)(K \cos^2\varphi - \mu \sin\varphi) \tag{8}$$

Using Eqs. (7) and (8) and the geometric relations

$$dx = \sin\varphi \, dS \tag{9}$$

and

$$dy = \cos\varphi \, dS \tag{10}$$

the cable's equilibrium configuration and tension are completely determined. It is also easy to make comparative studies between Glauert's model, which neglects the effects of skin friction, and our mathematical model which, of course, includes the effects of skin friction.

Nondimensionalize Eqs. (7–10) by defining the following functions:

$$\sigma = \frac{S}{T_0/K} \qquad \zeta = \frac{x}{T_0/K} \qquad \xi = \frac{y}{T_0/K}$$
$$\tau = \frac{T}{T_0} \qquad \eta = \frac{\mu}{K} \tag{11}$$

where T_0 is the tension in the cable at the point of connection to the towed vehicle. Hence its magnitude and direction (φ_0) are completely determined by the lift and drag on the towed vehicle.

Placing (11) into (7–10) we obtain

$$d\tau/d\sigma = (1/K)f(\varphi) + \eta \cos\varphi \tag{12}$$

$$d\varphi/d\sigma = (1/\tau)(\cos^2\varphi - \eta \sin\varphi) \tag{13}$$

$$d\zeta = \sin\varphi d\sigma \tag{14}$$

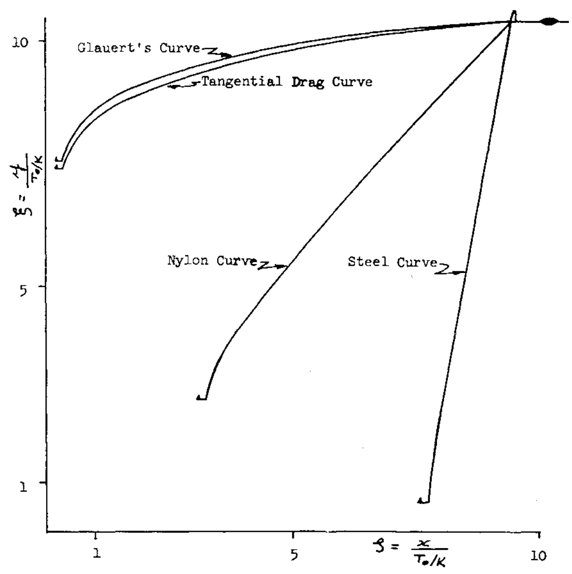

Fig. 3 Effect of cable weight and skin friction when body has negligible drag.

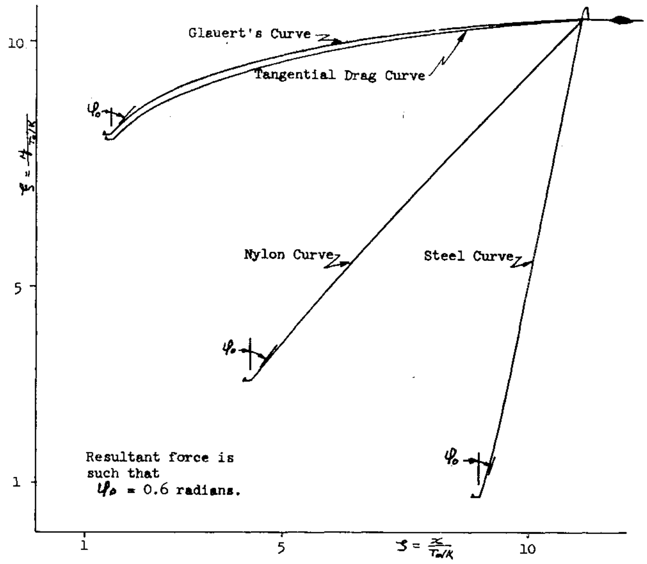

Fig. 4 Comparative study for a towed lifting body.

and

$$d\xi = \cos\varphi \, d\sigma \tag{15}$$

From our definitions of T_0 and φ_0 we have the necessary boundary conditions. Name y,

$$\tau_0 = T_0/T_0 = 1 \text{ at } \varphi = \varphi_0 \tag{16}$$

and $\varphi_0 = \varphi_{\text{initial}}$ (determined by the lift and drag on the towed vehicle).

With these boundary conditions, Eqs. (12) and (13) were integrated by computer using a Runge-Kutta scheme and the resulting curves for cable shapes and tensions were plotted.

Intuitively one would expect the tangential drag to increase with velocity and tend to force the cable to approach a straight line configuration in the limit. Hence, to illustrate the importance of the frictional force we have selected an extremely low velocity for our illustrative example. To this end experimental data was taken from Ref. 6, where lift and drag properties for a cable 1.22 in. in circumference were measured in a flow of 40 fps. Thus by a curve-fitting process (φ) was computed to be

$$f(\varphi) = 0\ 0134 \left(\frac{\varphi}{\pi}\right) + 0.0072 \left(\frac{\varphi}{\pi}\right)^2 - 0.0504 \left(\frac{\varphi}{\pi}\right)^3$$

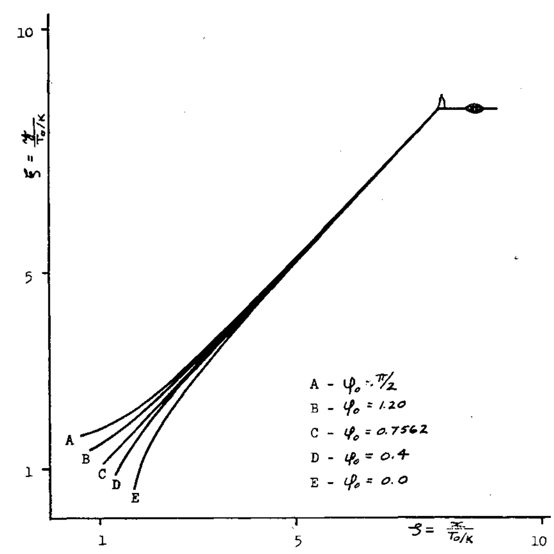

Fig. 5 Effect of lift-to-drag ratio on equilibrium configuration for a nylon cable.

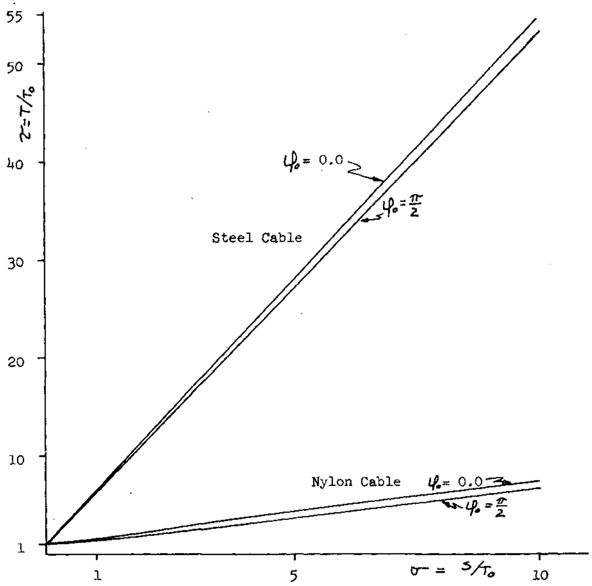

Fig. 6 Effect of cable weight and drag on tension in the cable.

Figure 3 contains comparative curves for the case where the drag of the towed vehicle is zero. That is, the special case when the towed vehicle is thrusting forward. The zero towed body drag produces a vertical tangent at the point of contact of towed vehicle and cable. Note that the two curves at the top are for the case when cable weight is neglected. This was done in an effort to focus complete attention on the effect of cable drag. Also note that Glauert's curve is an upper bound for all possible families of curves.

All curves in Figs. 3–6 were plotted using nondimensional parameters. In all cases, a relatively short cable length of $\sigma = 10$ was used in an effort to refrain from distorting the results. Figure 4 contains comparative curves for an arbitrary set of input data of drag and lift for the towed body, which results in an angle φ_0 at the cable-towed vehicle connection of 0.6 rad. Figure 5 contains a family of curves for nylon. These were plotted as a function of the change in lift and drag on the towed vehicle, hence changing φ_0. All other values were held constant. Figure 6 contains the extreme values of tension in the cable for a family of curves using nylon cables and steel cables. The nylon and steel, of course, represent extreme values in cable density.

Conclusion

The significant difference in the top two curves of Figs. 3 and 4 is entirely due to the effect of tangential cable drag, whereas the difference in the lower two curves is almost entirely due to cable density. Obviously, the density is a more important factor than the cable drag, although both are significant. Physically, when one eliminates both effects he, in essence, is making the assumption that the tension throughout the cable is a constant. Thus, there is but one possible cable configuration for the Glauert case. Figure 5 shows that this is an unreasonable assumption. Nevertheless, the Glauert case is important in that it represents the upper bound of all possible curves for a given set of flight conditions.

From Fig. 6 we see that the cable density is a more important constant than the flight attitude of the towed vehicle when determining cable tension.

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